Standards SI Prefixes

10^{-12}	10^{-9}	10^{-6}	10^{-3}	1	10^{3}	10^{6}	10^{9}	10^{12}
(p) pico	(n) nano	(μ) micro	(m) milli		(k) kilo	(M) mega	(G) giga	(T) tero

Electric Circuit

- Interconnection of electrical components that allow electrons to flow to perform a useful function
- Electric charge (q) is the electrical property of the atomic particles; measured in coulombs (C)
- Electrical current (i) is the the flow of charge (electrons) over time; measured in amperes (A)
- Voltage (V) is the potential difference in energy required to move a unit charge between two points; also known as the electromotive force – it is a relative difference between two points.
- Power (p) is the time rate change of energy which is generated (-) or absorbed (+); the net power in the system should be zero; measured in Watts (W)

Elements

Active: Generate Power

- Voltage sources
- Current sources (maintains a current)
- Generators

Passive: Absorb/dissipate power

- Resistors
- Capacitors
- Inductors

Passive Sign Convention

- Sign of the current relative to the voltage for a passive element
- We assume that the current enters the node at the higher voltage (+)
- In a wire, only one current flows at a time
- The current must enter the positive terminal
- Voltage polarity follows this convention
- The diagram and the math will dictate the correct direction
- Contradictions are solved with math

Voltage Notation

- Polarity is indicated by subscript notation
- First subscript indicates the higher-voltage node
- Second subscript indicates the lower-voltage node
- Often represented relative to the ground (in that case it is not a voltage difference)
- $V_{ab} = - V_{ba}$

Ohm's Law

- **Voltage = Current (I) x Resistance (R)**
- Resistance (R) is the ability to resist the flow of electric current; measured in ohms
- Resistors are either fixed or variable

- The voltage across a resistance is proportional to the current flowing through it
- Can be time varying
- Variable Resistor – Resistance can change without being removed (e.g. volume knob)
- Short-circuit – A circuit with an infinite current with resistance of zero.
- Open-circuit – A circuit that has infinite resistance (e.g. the wires are not connected so current can't flow)

Power

- **Power = voltage x current**
- Power is absorbed if power is positive
- Power is generated if the power is negative
- Determined by the PSC and common sense
- Resistors always absorb power
- If there is only one voltage source, it will always generate
- If more than one voltage source, the smaller power supply should absorb
- Voltage can absorb or deliver power

$P = V \times I$	$P = I^2 \times R$	$P = V^2 / R$
Active Elements Passive Elements	Passive Elements	Passive Elements

Parts of A Circuit

- Branch (b) - Elements; everything except the wire
- Node (n) - Point of connection between two or more branches
- Loop (l) - A closed path that never goes twice over a node
- Independent loop – It contains at least one branch which is not a part of any independent loop
- Max number of independent loops; $l = b - n + 1$

Kirchhoff's Current Law (KCL)

- The algebraic sum of the currents entering (or leaving) any node is zero; direction matters
- The sum of currents entering a node, is equal to that of the sum of currents leaving a node
- Based on the principle of conservation of charge
- Assume the signs of the currents and be consistent

Kirchhoff's Voltage Law (KVL)

- The algebraic sum of the voltages around any loop is zero; direction/polarity matters (PSC)
- Based on the conservation of energy
- Sum the increases and decreases of energy levels
- Make a loop, determine the direction of the current, and then apply the PSC

Single-Loop Circuits

- **Single Current but a different voltage**
- All elements are on one wire
- In-series/series elements have the same current within them
- In a single loop circuit, voltage across any resistor is proportional to its value
- Highest resistor = highest voltage

- The sum of all the voltages found from the resistors should be equal to the voltage that is supplied

Voltage Divider

- In a single loop circuit, we can find the voltage across any resistor using the voltage divider

$$v_{Ri} = \frac{R_i}{R_{eq}} v(t)$$

$$R_{eq} = R_1 + R_2 + \cdots\ R_N$$

- V_{Ri} = voltage across any resistor
- V(t) = Voltage source
- R_i = the targeted resistor (the resistor we would like to know the voltage of)
- R_{eq} = the sum/total resistance of all elements

Equivalent Resistance

- If multiple resistors are in-series, you can just add up the resistive elements
- The current flowing through the circuit will experience the same resistance
- If resistors appear in series (as a chain), they provide only one path for the current to take
- Do not forget that resistors have a voltage as well

$$i\ =\ \frac{V_s}{R_{Total}}$$

Equivalent Voltage Sources

- Use KVL to find the algebraic sum of voltage sources
- Use the sum of voltages to find an equivalent voltage source
- Voltage sources (supply) = Voltage of resistors
- Follow the PSC

$$V_{eq}\ =\ V_{R1}\ +\ V_{R2}\ +\ V_{Rn}$$

- We can replace all voltage supplies with 1 voltage source
- You can switch the location of any component without affecting anything
- Voltage sources in series add directly (algebraically) to form an equivalent voltage source
- Resistances in series add directly (direction doesn't matter) to form equivalent resistance

Strategy

1. Define a current.
2. Apply KVL to a single-loop circuit
3. Apply Ohm's law to each voltage
4. Add up the voltages to find current
5. The current is the same in each wire
6. Knowing the current, we can individually solve for the voltage for each element

Note: When finding voltages within a single loop, only add up the elements for THAT specific loop (to not waver outside of that loop)

Single Node-Pair Circuits (Parallel)

- **Elements that have the same voltage across them**
- **Current is different in each element**
- Parallel elements
- Resistors are in parallel if they share the same two nodes (same voltage)
- Total resistance should be smaller than the smallest resistance value in the circuit

$$R_p = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_N}}$$

- R_p = Sum of total resistance in a parallel circuit

Special Cases

1. $R_p = \frac{R_1 \times R_2}{R_1 + R_2}$; if there are only two resistors in parallel

2. $R_p = \frac{R}{N}$; if there are N parallel resistors with equal values of R

Current Divider Equation
- In a single node circuit with multiple resistors and current source, we can find any current
- Find the current found in any resistor

$$i_k = \frac{R_p}{R_k} i_0$$

- I_k = current through the targeted resistor
- R_k = Targeted resistor
- R_p = Total resistance in a parallel circuit
- I_0 = Current source

Special Case: Current Divider

$$i_1 = \frac{R_2}{R_1 + R_2} \times i_0$$

$$i_2 = \frac{R_1}{R_1 + R_2} \times i_0$$

- Use these equations to find the current in a circuit with only **two parallel resistors**
- **THE RESISTOR ON TOP IS OPPOSITE TO THE CURRENT THAT YOU WANT**

Equivalent Current Sources
- In a parallel circuit, you can add up current sources (algebraically)
- You are able to use KCL (current entering = current exiting)
- Rely on the definition of a parallel circuit over the appearance of the circuit
- Rely on the sign of the net value to determine the direction

Series and Parallel Resistor Combinations
- Find a pair of resistors

- Determine if they are in-series or parallel
- Parallel – Node pairs (Find special cases or use equations)
- In-series – Same current (Add up resistors)
- Use these definitions to simplify a circuit
- Use the special cases of resistors to your advantage

Strategy

1. Apply resistance equations
2. Apply current divider equations
3. Know that voltage is the same throughout the elements

Multi-Nodal Analysis

- To determine the voltages in a circuit
- Nodal Analysis uses **Kirchhoff's Current Law as well as Ohm's Law** to find the voltages in a circuit
- Recall that any current entering must exit
- You can use the current divider!

1. Set a reference (usually the ground); Any node voltage is reference to the ground unless otherwise defined
2. Identify all known node voltages
3. At each node with an unknown voltage, write a KCL equation
4. Replace all currents in terms of node voltages
5. Solve for unknown voltages
6. Use voltages to solve for any desired values

REMEMBER THAT ALL VOLTAGES ARE DIFFERENCES WHEN APPLY OHMS LAW.

$$I = \frac{\Delta V}{R}$$

Multi-Loop Analysis

- More loops than one basic loop
- Also called the Mesh Analysis
- Uses **Kirchhoff's Voltage Law and Ohm's Law** to find the currents
- When there are more Nodes than Loops, apply this analysis.

Loop: A closed path that does not go over any node twice.

Mesh: A loop that does not enclose any other loop.

Loop Current: A (fictitious) current that is assumed to flow around a loop. Fictitious as it is more of an intermediate step. We want to know the current that is flowing through an element. You can add fictitious currents up together.

Mesh Current: A loop current associated with a mesh.

Always try to make the least number of loops. (Recall $l = b - n + 1$).

1. Identify all mesh's and assign mesh current
2. Identify all known currents
3. For each loop with an unknown current, write a KVL equation
4. Replace voltages in terms of mesh currents (V = IR)
5. Solve for mesh currents
6. Solve for unknown currents
7. Use the current to solve for the values needed

Shortcut in determining loops and their current:

1. Subtract resistors that are a part of both meshes with the opposite mesh that you are writing the equation for.

Example:

Loop 1:

$$i_1(R_1 + R_2 + R_3) - R_3 i_2 \ = \ V_{s1}$$

From this equation, we can recognize there are three resistors (R1, R2, R3) in Loop 1. However, R3 is also a member of Loop 2. Therefore, we can subtract it (with the opposite current).

Loop 2:

$$i_2(R_3 + R_4 + R_5) - R_3 i_1 \ = \ -V_{s2}$$

Again, from this equation we can recognize there are three resistors (R3, R4, R5) in Loop 2. However, R3 is also a member of Loop 1. Therefore, we can subtract it (with the opposite current).

ALWAYS TAKE INTO ACCOUNT OF THE PASSIVE SIGN CONVENTION.

You will now have a system of equations.
Remember to place your desired currents as an addition of the mesh currents you found!

Thevenin

- Replace a complicated circuit with a simple one such that the load cannot tell the difference
- We replace the circuit with the same voltage-current characteristics (As long as the voltage and current stays the same).
- Replaces the linear two-terminal circuit with a voltage source in series with a resistance
- Redesigning your circuit is earlier in simple circuits

1. Identify and isolate the circuit terminals
2. Eliminate the independent sources and determine the equivalent R_{TH} of the circuit (R_{TH} is the "sum" of all the resistors)
3. Determine the open-circuit voltage V_{OC} across circuit terminals (This will be V_{TH})
4. Place the TH equivalent circuit into the original circuit and preform the desired analysis

Current Source – Open Circuit

Voltage Source – Short Circuit

Norton

- Replace a complicated circuit with a current source, with a short-circuit current through the terminals

1. Isolate the terminals for where the equivalent circuit is desired.
2. Eliminate the sources and determine R_{TH} of the circuit
3. Re-activate the sources, short-circuit the output terminals and determine the current
4. Place the Norton equivalent circuit into the original circuit and perform the desired analysis

Current Source – Open Circuit
Voltage Source – Short Circuit

Maximum Power Transfer

Thevenin circuit is useful to find the maximum power transferred to a load
In short:

$$P_{max}\ is\ when\ R_{TH} = R_L$$

Source Transformation

- We can solve for i in the TH equivalent
- We can find V in the N equivalent
- The Thevenin and Norton circuits are connected by Ohms law
- Any voltage source in series with resistance can be modelled as a current source in parallel with the same resistance and vice-versa (You can transform series to parallel)
- The goal in source transformation is to create a purely series or purely parallel circuit

WHEN FINDING THE EQUIVALENT VOLTAGE OR CURRENT, DO NOT REMOVE THE RESISTORS. THEY ARE EVALUATED ON THEIR OWN (TOGETHER). RESISTORS CAN ONLY BE “REMOVED” BY COMBINING THEM WITH OTHER RESISTORS.

TLDR: RESISTORS ONLY REMOVED WHEN “ADDED”, NOT DURING OHM’S LAW.

1. Use Ohm’s law to combine resistors with voltage to get current – or vice-versa.
2. Combine all the resistors accordingly to create an equivalent total resistance
3. Combine current sources or voltage sources to find an equivalent source (POLARITY MATTERS)

WHEN COMBINING RESISTORS, YOU CANNOT TRANSFORM A COMPONENT IN WHICH YOU NEED A CURRENT.

4. Use either voltage divider, or current divider to preform your analysis

Superposition

- To determine the output response from each source (response can mean current or voltage)
- Eliminate ALL other sources (except one) and analyze the resulting circuit

- Repeat the process for reach source

VOLTAGE SOURCE – SHORT CIRCUIT
CURRENT SOURCE – OPEN CIRCUIT

REMOVE ONLY ONE AT A TIME.

1. Eliminate current sources to find a mesh and current across the elements
2. Eliminate voltage source to find current across elements
3. Sum the contributions of responses from ALL sources
4. You MUST write the real currents as a sum of the mesh currents
5. Use either current or voltage divider to preform analysis

Complex Numbers

- Radians is a way to measure the arc of the circle
- Sinusoids – Has the form of a sine or cosine function

$$v(t) = V_m \sin(\omega t + \varphi)$$

V_m = Amplitude
f = Signal frequency (How fast the ROC)
$\omega = 2\pi f$ = Angular frequency (rad/s)
$T = \frac{1}{f}$ = Signal period (length of time for 1 cycle)
φ = Phase (shifts the function)

Complex Numbers:
- Real part and then imaginary
- Z = x + jy
- Treat as a vector with an imaginary j-axis

$$j = \sqrt{-1}, where \left(\sqrt{-1}\right)^2 = -1$$

- Thus, magnitude can still be found
- Treat as another variable
- You can add and subtract and multiply an divide
- You can multiply by the conjugate

Frequency Response

- Frequency Response: Variation in its behaviour with change in signal frequency
- Resistance (R): The friction against the flow of current
- Reactance (X): The inertia against the flow of currents (frequency dependent); capacitors and inductors

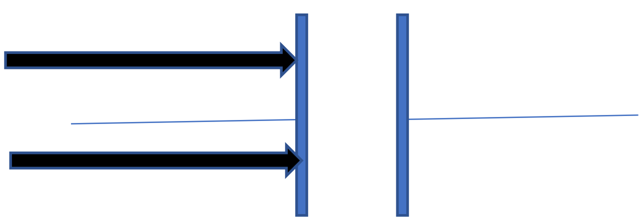
IMPEDANCE (Z):

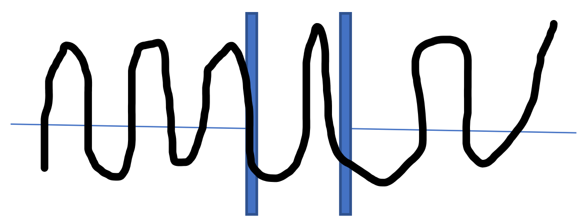
- The expression of any and all forms of opposition to current flow
- Includes resistance and reactance
- R is real and X is imaginary
- Frequency dependent
- Measured in Ohms (Ohm's law still holds) - Passive
- Combined using the same rules developed for resistors
- KVL and KCL laws hold for impedance

$Z = R + jX$ and $Z = \sqrt{R^2 + X^2}$

CAPACITORS

- Ability to store charge
- The ratio of the charge q to the voltage difference "v" between the two plates
- Measured in farads (F)
- Passive
- Ideal capacitor does not dissipate energy – it stores energy in its field and returns it as power
- Open Circuit at 0 Hz (DC), Short Circuit at infinite Hz (AC)





DC will not pass.
Open Circuit

AC will pass.
Short Circuit.

VARYING FREQUENCY AND CAPACITANCE:

$X_c = \frac{1}{\omega C}$ - Reactance

$Z_c = -jX_c$ - Impedance

$$Z_C = \frac{1}{j\omega C}$$

Total Impedance:

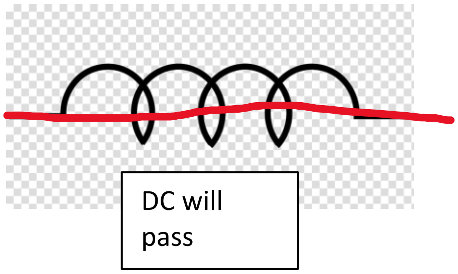
$$Z = R + \frac{1}{j\omega C}$$

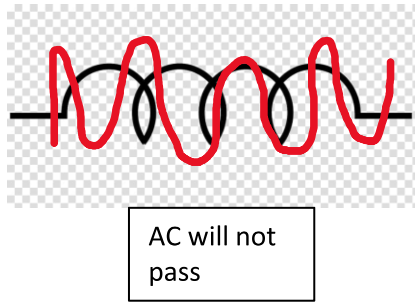
Usually, all values are given to you. Recall that total impedance has a "real" part and "imaginary" part

INDUCTORS

- A passive element designed to store energy in its magnetic field

- Exhibits opposition to the change of current
- Measured in Henry (H)
- Opposite properties of a capacitor
- Can be combined with the same rules as resistors
- Short Circuit for 0 Hz (DC), Open Circuit for infinite Hz





VARYING FREQUENCY AND INDUCTORS
$X_L = \omega L$ - Reactance
$Z_L = jX_L$ - Impedance

$Z_L = j\omega L$

Total Impedance:

$$Z = R + j\omega L$$

Usually, all values are given to you. Recall that total impedance has a "real" part and "imaginary" part.

Recall that magnitude is only the "real" parts.

Decibels

Logarithm:
- Gives the exponent values
- Usually log base 10

Frequency Response: The variation in the behaviour of a circuit with a change in signal frequency.

Transfer Function: A frequency dependents ratio of output (response) to the input (source) of a circuit
- Typically given as a ratio of given voltages
- Comparing output voltage to input voltage

$H(\omega) = \frac{V_o(\omega)}{V_s(\omega)}$ = out / in

- Find Vo in terms of Vs using a voltage divider
- We don't need to have values – all we need is a ratio
- It is a ratio for any arbitrary set of values

Decibels

- To account for a non-linear relationship between this ratio we use dB
- It is a logarithmic way of describing a ratio
- The ratio may be sound pressure, intensity, power, etc...
- Compresses the higher range, and expresses the lower range
- Plotted using a Bode plot
- Uses the logarithmic axis for frequency and decibels for magnitude

$$\left(\frac{V_o}{V_s}\right)_{dB} = 20\log_{10} \left(\frac{V_o}{V_s}\right)$$

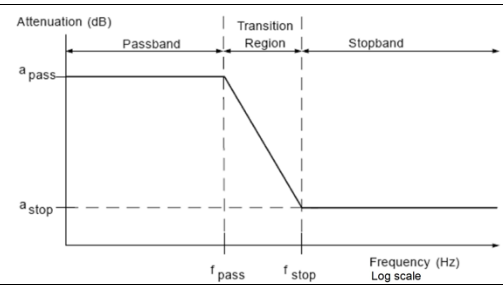
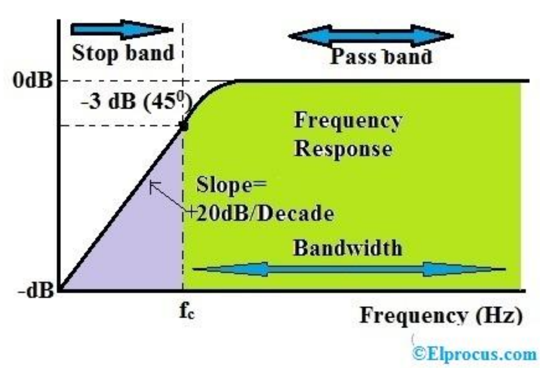
Bode Plot

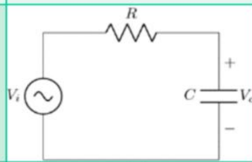
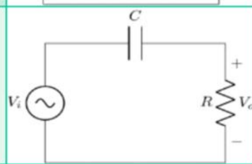
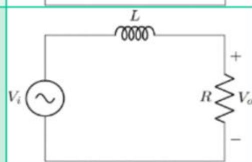
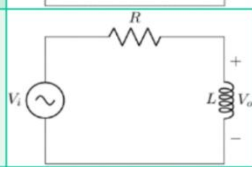
- Given a certain transfer function, you can find the amplitude [V/V] using certain frequencies
- With your amplitude [V/V], you can convert it into dB
- When plotting or reading Bode plots, the scale is VERY important
- Normally, you should count back from the highest graduation to find your value of choice
- Be able to transfer from dB to Amplitude (undo the logarithm)
- You must also be able to read the graph and find the Y-value (dB) associated with it

Filters

- Passive Filter – One that contains R, L, and C components
- Active Filter – Along with R, L, C, it also contains an energy source (Not in this course)

Types of Filters:

Type	Description	Photo
Low pass	Passes frequencies from DC (0 Hz) to cut off (fc)	
High pass	Passes frequencies from cut off (fc) to infinity	

Filter Type	Typical Circuit	Transfer Function	Cut-off Frequency	Bandwidth
RC LPF		$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\mathbf{Z}_c}{R + \mathbf{Z}_c}$	$f_c = \frac{1}{2\pi RC}$	$BW = 0 \text{ Hz to } f_c$
RC HPF		$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + \mathbf{Z}_c}$	$f_c = \frac{1}{2\pi RC}$	$BW = f_c \text{ to } \infty$
RL LPF		$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + \mathbf{Z}_L}$	$f_c = \frac{R}{2\pi L}$	$BW = 0 \text{ Hz to } f_c$
RL HPF		$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\mathbf{Z}_L}{R + \mathbf{Z}_L}$	$f_c = \frac{R}{2\pi L}$	$BW = f_c \text{ to } \infty$

Bode Plot:

1. Use the equation you found for the transfer function and convert values of angular frequency to amplitude [V/V]
2. Convert [V/V] to decibels

Ratio:
- Transfer function value for the cut off function is 0.7071
- If the ratio for the transfer function is 1, then Vo = Vi
- Filters not in series will always have a roll-off rate for –20 dB
- Filters can be cascaded (arranged in series) to increase the filter (higher-order)

Bandpass Filters
- Passes a range of frequencies from a lower cut-off frequency to an upper cut of frequencies
- Bandwidth – Width of the passband
- Lower cut-off frequency (fL) - Passband and lower stopband
- Upper cut-off frequency (fH) - Passband and upper stopband
- Center frequency (fo) - Center of the passband where magnitude is the max
- Quality factor (Q) - The selectivity of a bandpass filter

Cascaded Systems (In series)
- A bandpass filter can be formed by putting a lowpass and highpass filter in series (cascading)
- You are essentially multiplying the bode plots
- High-pass filter sets the lower cut-off frequency
- Low-pass filter sees the upper cut-off frequency

1. Separate the system into blocks
2. Determine which block has a lowpass filter and which has a highpass filter

3. Create a transfer function for both of them
4. H = H1 x H2

$$mag\left\{\frac{V_0}{V_s}\right\} = H_1 \times H_2$$

- Makes analysis easier but we want to see there interactions together so should instead use their cut-off frequencies

$f_H = \frac{1}{2\pi R_{low} C_{low}}$; use the R and C in the lowpass block for the highpass – ALWAYs higher than fL

$f_L = \frac{1}{2\pi R_{high} C_{high}}$; use the R and C in the highpass block for lowpass

$$(BW) = f_H - f_L$$

$f_o = \sqrt{f_L \times f_H}$; Center frequency

$$Q = \frac{f_o}{BW}$$