REVIEW ON BOOLEAN LOGIC AND ALGEBRA

ECOR 1044 – Mechatronics

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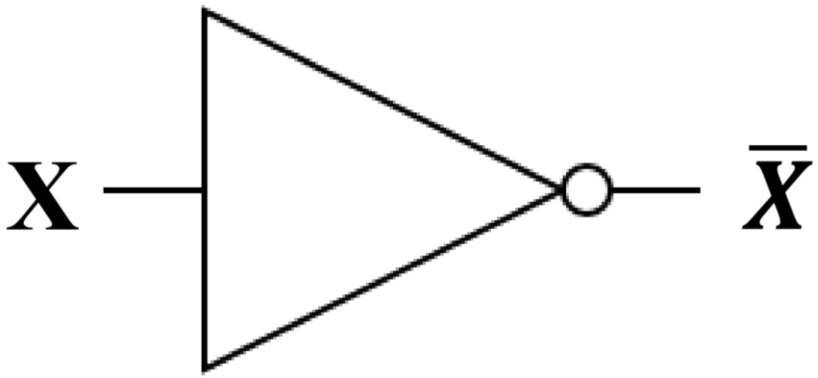
BOOLEAN LOGIC AND ALGEBRA

- VALUES ARE: 1, 0, True, False, High, Low
- VARIABLES CAN ONLY BE VALUES

COMPLEMENT, NOT, INVERSE

$\overline{X}$ is the opposite of x

X	$\overline{X}$
0	1
1	0

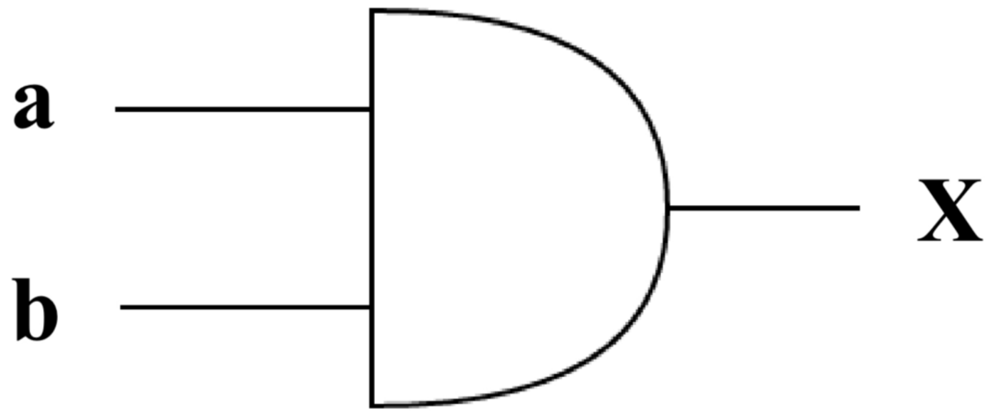


AND

ab OR $a \cdot b \longrightarrow \cdot$

X is 1 if a and b are <u>both</u> 1.

a	b	x
0	0	0
1	0	0
0	1	0
1	1	1

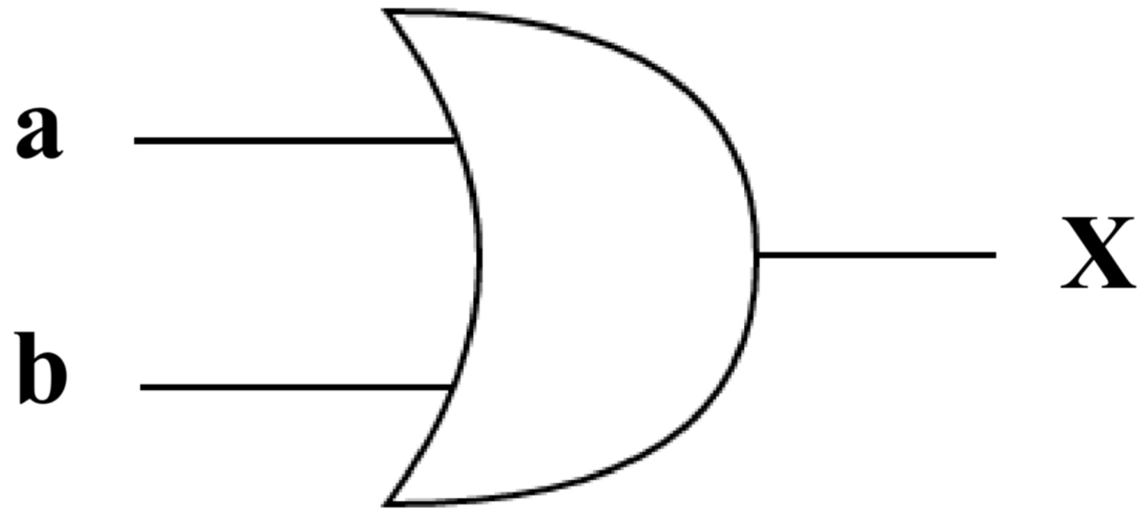


OR

$a + b \longrightarrow +$

X is 1 if a or b or both are 1

a	b	x
0	0	0
1	0	1
0	1	1
1	1	1



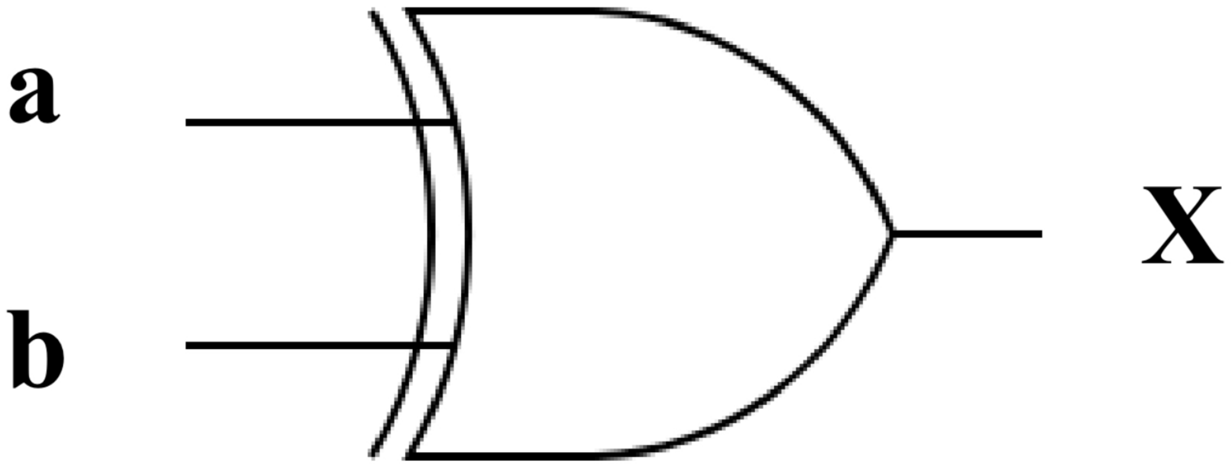
*At least one variable is 1

XOR (Exclusive OR) $a \otimes b$

X is 1 if exactly one of a or b is 1.

a	b	X
0	0	0
1	0	1
0	1	1
1	1	0

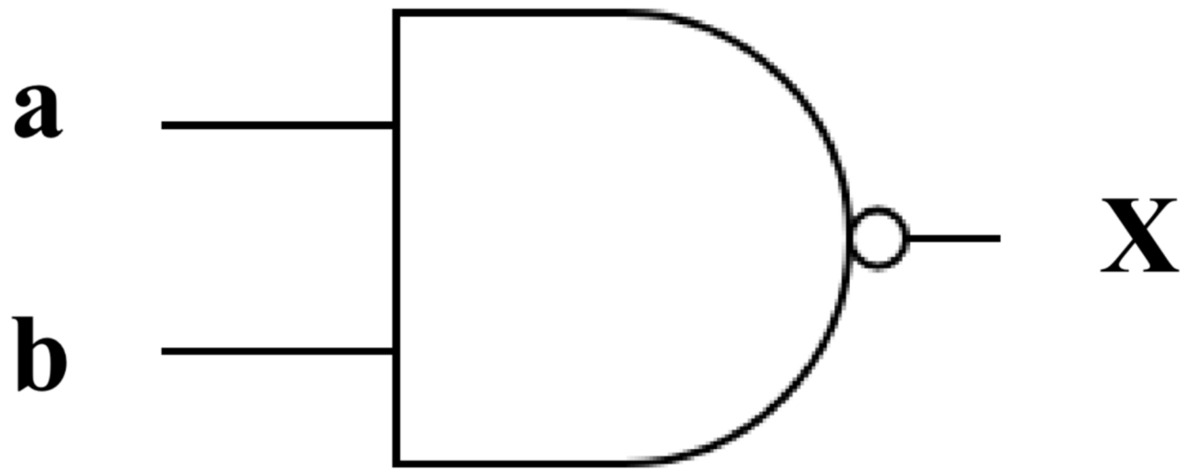
$a \mathrel{!=} b$



NAND $\overline{ab}$ OR $\overline{a \cdot b}$

X is 0 if a and b are both 1

a	b	x
0	0	1
1	0	1
0	1	1
1	1	0

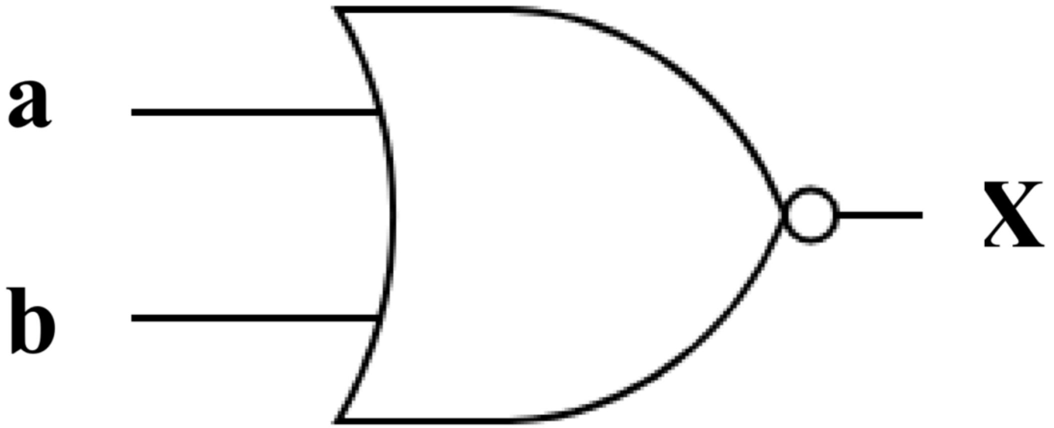


NOR

$\overline{a + b}$

X is 0 if a or b or both are 1

a	b	x
0	0	1
1	0	0
0	1	0
1	1	0



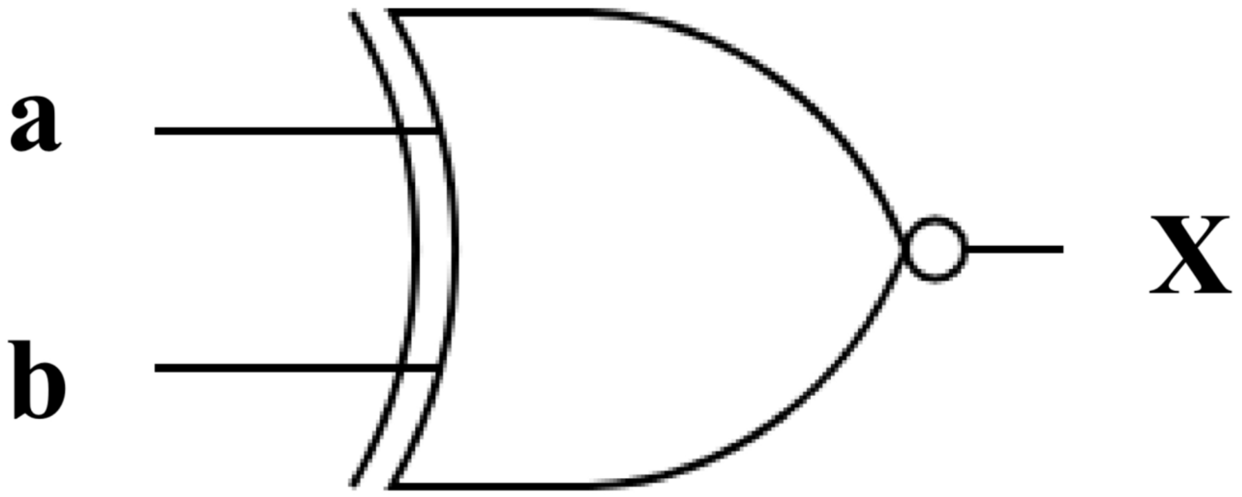
X-NOR

$\overline{a \oplus b}$

X is 0 if exactly one of a or b is 1

a	b	X
0	0	1
1	0	0
0	1	0
1	1	1

$a \mathrel{!=} b$



BOOLEAN ALGEBRA LAWS

$F = AB + BC' + AC$

Complemented Variable

Redundancy term

The four “BASIC” laws:

1. X or 0 equals X

$$X + 0 = X$$

2. X or 1 equals 1

$$X + 1 = 1$$

3. X and 0 equals 0

$$X \,.\, 0 = 0$$

4. X and 1 equals X

$$X \,.\, 1 = X$$

BOOLEAN ALGEBRA LAWS

INDEMPOTENT LAWS:

1. X or X = X

   X + X = X

2. X and X = X

   X . X = X

3. X or not X = 1

   X + /X = 1

4. X and not X = 0

   X . /X = 0

INDEMPOTENT LAWS:

A variable is unchanges when operating on itself.

Double Negative:

5. Not not X = X

   /(/X) = X

BOOLEAN ALGEBRA LAWS

Communative:

1. $X + Y = Y + X$

2. $X . Y = Y . X$

Assosiative

1. $X + (Y + Z) = (X + Y) + Z$

2. $X . (Y . Z) = (X . Y) . Z$

Distributive (Common with two uniques)

1. $(X + Y) . Z = (X . Z) + (Y . Z)$

2. $(X . Y) + Z = (X + Z)(Y + Z)$

BOOLEAN ALGEBRA LAWS

Absorption Law

$A + AB = A$

$A . (A + B) = A$

Absorption Law (Inverse)

$A + /A . B = A + B$

$A . (/A + B) = AB$

DUALITY PRINCIPLE

You can take any valid Boolean identity and find it’s dual

1. Place brackets around all AND terms
2. Substitute all . With +
3. Substitute all 0 with 0

This is another valid identity. They are DUALS. Not equals.

If you remember one identity, you know the other.

Gate Formulas From Truth Tables

1. Write out the Truth Table
2. Look where the result is “1”
3. Write the letters with the corresponding to the inputs
4. AND terms
5. Equation is the OR of all terms where the output is “1”

XOR Properties

1. Inverting any of the leads changes the XOR gate into an XNOR gate
2. Making one input a static (never changing) “1” creates an inverter
3. Can be a controlled inverted
4. Can be an equality check
5. Can check the number of inputs (even: 0, odd: 1)

DeMorgan's Law

General DeMorgan's Law

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

inverse

$$\overline{\overline{A} + \overline{B}} = A \cdot B$$

↳ OR gate inverted = AND gate inverted
↳ AND gate inverted = OR gate inverted

↳ Inverting in/out of OR → AND
↳ Inverting in/out of AND → OR

Dual

$$\overline{D + E} = \overline{D} \cdot \overline{E}$$

inverse

$$\overline{\overline{D} \cdot \overline{E}} = D + E$$

Break/Join bar and switch operation.

1. Boolean or input
2. Brack ANDs
3. Take the dual:
   - ↳ AND → OR, OR → AND
   - ↳ Clean-up brackets
4. Invert all variables

Be careful!!!!!!!!!!

F → The original boolean expression.

$\boxed{F_{dual}}$

↳ Interchanging
AND → OR ,
$X \rightarrow \overline{X}$

↳ Think of the basic laws of boolean logic

$F \neq F_{dual}$

$\boxed{\text{De Morgan's Laws}}$

↳ How to deal with NOT

1. $\overline{A \cdot B} = \overline{A} + \overline{B}$
2. $\overline{A + B} = \overline{A} + \overline{B}$

↳ Breaking / joining of the bar

* By applying DeMorgan's law you can find an <u>equivalent</u> expression to F.

$F = F_{DeMorgan}$

* Usually done by applying DeMorgan's law + back substitution.

Karnaugh Maps

- Another way of representing a truth table
- Each square on the map represents an output for a different input combinations (with all possible input combinations represented)
- Any single square represents a value for # of inputs
- Two adjacent squares will always have one common variable
- You may loop inputs in powers of 2

Special Case: Karnaugh Map

- Static output of 1:
  - When all of the outputs are 1, F = 1

BC \ A	0	1
00	1	1
01	1	1
11	1	1
10	1	1

$$F = A \cdot \bar{A} + B \cdot \bar{B} + C \cdot \bar{C}$$
$$= 1 + 1 + 1$$
$$F = 1$$

Between Two Loops
↳ OR

Between Same Loop
↳ AND